



# ADVANCED MATHEMATICS

Final Exam - December 2013

Name: \_\_\_\_\_

NIU: \_\_\_\_\_ Group: \_\_\_\_\_

Grade: \_\_\_\_\_

**Instructions:** This exam consists of six questions. You have two hours to give a reasoned answer to all the exercises. Write the quiz entirely in ink. Calculators are not permitted.

- 1 Determine for which values of the parameter  $a \in \mathbb{R}$  the matrix  $A$  is diagonalizable. (20 points)

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & a \end{pmatrix}$$

2] Suppose that in a given market with a single commodity the demand function is  $D(P) = 4 - P$  and the supply function is  $S(P) = -2 + 2P$ , where  $P > 0$  denotes the unitary price of the good. Assume that time is discrete and that the market follows the dynamics of the Cobweb Model, that is,  $S(P_t) = D(P_{t+1})$  for every  $t$ . (15 points)

- (a) Obtain the expression of  $P_t$  when  $P_0 = 4$ .
- (b) Calculate the equilibrium  $\bar{P}$ .
- (c) Analyze the behavior of the price in the long run.

3 Consider the following system of equations (20 points)

$$X_{t+1} = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix} X_t.$$

- (a) Obtain the solutions of the previous system.
- (b) Calculate the equilibrium  $\bar{X}$ .
- (c) Is the equilibrium  $\bar{X}$  globally asymptotically stable?

4 Solve the following differential equation:  $(t^2 - 1)x' = -2tx$ . (15 points)

5 Solve the following equation:  $x'' - 5x' + 6x = e^{2t}$ . (20 points)

6 Consider the differential equation  $x' = f(x)$ . The following picture shows the trajectories of the solutions of such an equation. (10 points)

- (a) Identify the equilibria.
- (b) Study the stability of those equilibria.
- (c) Draw a sketch of the phase diagram that would correspond to this situation.

